Interference effects on the coupling impedance of many holes in a coaxial beam pipe

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The problem of many holes in a coaxial beam pipe is studied by means of the modified Bethe theory. The electromagnetic fields propagating in the coaxial region couple the equivalent dipole moments of the holes. The effect of the coupling on the longitudinal impedance and on the loss factor is investigated, showing that the interference phenomena are significant for such geometries. [S1063-651X(97)08110-5]

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I. INTRODUCTION

In this paper we study the coupling impedance and the loss factor of coaxial structures with multiple pumping holes.

The analytical solution of a many-hole problem has been given in the case of a circular beam pipe with thick walls [1], the method being based on Bethe's diffraction theory. The longitudinal impedance is calculated from the coherent sum of the fields generated by each hole.

The impedance of a single hole in a coaxial structure has been calculated numerically [2] and analytically applying Bethe's modified theory [3] and by variational methods [4]. The results obtained with these different procedures show a good agreement.

In this paper we extend Bethe's modified theory to the general case of N holes in an infinitely long perfectly conducting coaxial pipe (Fig. 1). The reaction fields have to be considered in order to fulfill the energy conservation law. We evaluate the effect of the interference of the fields generated by the equivalent dipoles taking into account also the coupling among the dipoles. The self-consistent solution shows that the coupling between holes can affect significantly the radiated energy spectrum and the coupling impedance. The reaction fields introduce in fact a coupling between the equivalent dipole moments of different holes.

In Sec. II we outline Bethe's modified theory applied to the calculation of the longitudinal impedance. Impedance and loss factor are treated in Sec. III. Finally, in Sec. IV, we compare our results to those obtained with the MAFIA simulation code.

II. GENERAL THEORY

The general theory adopted in our calculation is described in [3,5]. For the sake of convenience, we summarize its important features at frequencies below the beam pipe cutoff considering only scattered TEM-type fields.

Bethe's diffraction theory states that each hole is equivalent to an electric and a magnetic dipole whose moments are given by

$$M_\varphi(z_i)=\alpha_m[H_{0\varphi}(z_i)-H_{s\varphi}(z_i)],$$

$$P_r(z_i)=\varepsilon\alpha_e[E_{0r}(z_i)-E_{sr}(z_i)], \tag{1}$$

where α_m and α_e are the hole polarizabilities and $H_{s\varphi}$ and E_{sr} are the scattered fields calculated at the hole center. The primary magnetic and electric fields, generated by a point charge q, traveling with velocity c along the axis of a perfectly conducting pipe, are

$$H_{0\varphi}(z_i)=H_{0\varphi}(0)e^{-jk_0z_i},\quad E_{0r}(z_i)=E_{0r}(0)e^{-jk_0z_i}, \tag{2}$$

with

$$E_{0r}(0)=Z_0H_{0\varphi}(0)=Z_0\frac{q}{2\pi b}. \tag{3}$$

In general the scattered fields can be expressed as a superposition of modes. The coefficients of the modal expansion are determined through the Lorentz reciprocity principle [5]; they are linear functions of the equivalent dipole moments of the apertures which can be obtained solving a $2N\times 2N$ sized linear system.

Once the equivalent dipole moments have been determined, using the definition of the longitudinal impedance [6]

$$Z(\omega)=-\frac{1}{q}\int_{-\infty}^{+\infty}E_z(r=0)e^{jk_0z}dz, \tag{4}$$

it is straightforward to derive a general expression of the longitudinal impedance for N holes centered in $z=z_i$,

$$Z(\omega)=j\frac{\omega Z_0}{2\pi qb}\sum_{i=1}^{N}\left[\frac{1}{c}M_\varphi(z_i)+P_r(z_i)\right]e^{jk_0z_i}. \tag{5}$$

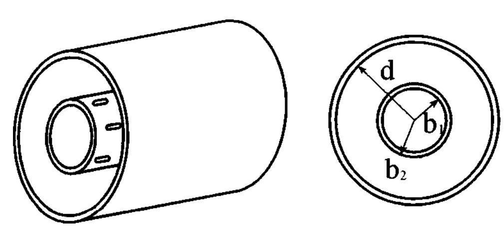

FIG. 1. Relevant geometry.

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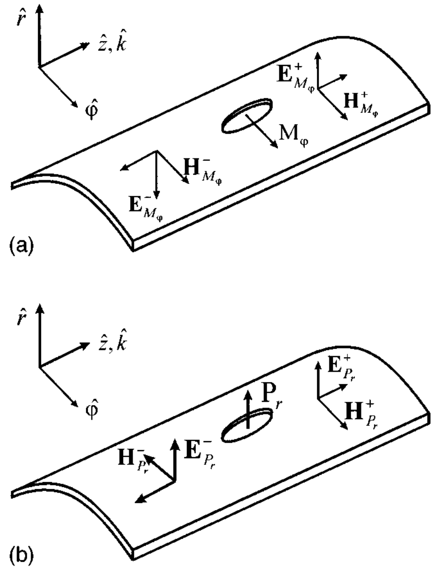


FIG. 2. (a) TEM field generated by an equivalent magnetic dipole moment M_φ. (b) TEM field generated by an equivalent electric dipole moment P_r.

III. HOLES IN A COAXIAL PIPE

Each dipole moment radiates a forward and a backward wave along the coaxial pipe. While the waves produced by the electric and magnetic dipole have the same phase along the beam direction, they are in phase opposition along the other (Fig. 2).

Using the expressions of the fields generated by the dipoles (Appendix), we can therefore write Eqs. (1) as

$$M_\varphi(z_i)=\alpha_m\left[H_{0\varphi}(z_i)-j\,\frac{\omega}{2}\,\mu h_{0\varphi}^2\sum_{h=1}^{N} M_\varphi(z_h)e^{-jk_0|z_h-z_i|}\right.$$
$$\left.+j\,\frac{\omega}{2}\,h_{0\varphi}e_{0r}\sum_{h=1}^{N} P_r(z_h)\mathrm{sgn}(h-i)e^{-jk_0|z_h-z_i|}\right], \tag{6}$$

$$P_r(z_i)=\varepsilon\alpha_e\left[E_{0r}(z_i)-j\,\frac{\omega}{2}\,e_{0r}^2\sum_{h=1}^{N} P_r(z_h)e^{-jk_0|z_h-z_i|}\right.$$
$$\left.+j\,\frac{\omega}{2}\,\mu h_{0\varphi}e_{0r}\sum_{h=1}^{N} M_\varphi(z_h)\mathrm{sgn}(h-i)e^{-jk_0|z_h-z_i|}\right], \tag{7}$$

having indicated with e_{0r} and $h_{0\varphi}$ the normalized modal function for the TEM mode.

Equations (6) and (7) can be summarized as

$$\begin{pmatrix} a_{ih} & \alpha_m b_{ih} \\ \frac{\alpha_e}{c^2}\,b_{ih} & c_{ih}\end{pmatrix}\begin{pmatrix}\mathbf{M}_\varphi(z_i)\\ \mathbf{P}_r(z_i)\end{pmatrix}=\begin{pmatrix}\alpha_m\mathbf{H}_{0\varphi}(z_i)\\ \varepsilon\alpha_e\mathbf{E}_{0r}(z_i)\end{pmatrix}$$
$$(i,h=1,2,\ldots,N), \tag{8}$$

where $\mathbf{H}_{0\varphi}=(H_{0\varphi}(z_1),\ldots,H_{0\varphi}(z_N))$, $\mathbf{E}_{0r}=(E_{0r}(z_1),\ldots,E_{0r}(z_N))$, similarly for $\mathbf{M}_\varphi$ and $\mathbf{P}_r$, and

$$a_{ih}=j\,\frac{\omega}{2}\,\alpha_m\mu h_{0\varphi}^2e^{-jk_0|z_i-z_h|}+\delta_{ih},$$
$$b_{ih}=\mathrm{sgn}(i-h)j\,\frac{\omega}{2}\,h_{0\varphi}e_{0r}e^{-jk_0|z_i-z_h|},$$
$$c_{ih}=j\,\frac{\omega}{2}\,\alpha_e\varepsilon e_{0r}^2e^{-jk_0|z_i-z_h|}+\delta_{ih}, \tag{9}$$

δ_{ih} being the Kronecker symbol.

System (8) can be solved directly by inversion of the coefficients' matrix or by some iterative procedure. Since we are interested in the low frequency behavior of the impedance below the cutoff of the $\mathrm{TE}_{1,1}$ mode, we can limit ourselves to the first step of the iterative procedure, that is, replacing the electric and magnetic dipole moments in the right-hand side of Eqs. (6) and (7) with their approximated values

$$M_\varphi(z)=\alpha_mH_{0\varphi}(z)\ \text{ and }\ P_r(z)=\varepsilon\alpha_eE_{0r}(z) \tag{10}$$

from which we derive the low frequency approximation for the longitudinal impedance

$$Z(\omega)=jZ_0\,\frac{k_0}{4\pi^2b^2}\left[N(\alpha_m+\alpha_e)-\frac{k_0}{4\pi b^2\ln(d/b)}\right.$$
$$\left.\times(\alpha_m-\alpha_e)^2\sum_{h=1}^{N-1}\sum_{w=1}^{N-h}\,\sin\left(2k_0\sum_{t=1}^{w}\,l_{h+t}\right)\right]$$
$$+Z_0\,\frac{k_0^2}{16\pi^3b^4\ln(d/b)}\left[\frac{N^2}{2}\,(\alpha_m+\alpha_e)^2\right.$$
$$+\frac{N}{2}\,(\alpha_m-\alpha_e)^2+(\alpha_m-\alpha_e)^2\sum_{h=1}^{N-1}\sum_{w=1}^{N-h}$$
$$\left.\times\cos\left(2k_0\sum_{t=1}^{w}\,l_{h+t}\right)\right], \tag{11}$$

with $l_h=z_h-z_{h-1}$.

For N equally spaced holes Eq. (11) yields

$$Z_{\mathrm{Re}}(\omega)=Z_0\,\frac{k_0^2}{32\pi^3b^4\ln(d/b)}\left\{N^2(\alpha_m+\alpha_e)^2\right.$$
$$\left.+(\alpha_m-\alpha_e)^2\left[\frac{\sin^2(Nk_0l)}{\sin^2(k_0l)}\right]\right\} \tag{12}$$

and

$$Z_{\mathrm{Im}}(\omega)\approx Z_0\,\frac{Nk_0}{4\pi^2b^2}\,(\alpha_m+\alpha_e) \tag{13}$$

having neglected the frequency higher order term in the imaginary impedance. It is worth noting that the imaginary impedance of N holes is, in first approximation, independent of the holes' position, equal to N times the impedance of a single hole. The real part oscillates between

$$N^2 \quad \text{and} \quad \frac{(\alpha_m+\alpha_e)^2}{2(\alpha_m^2+\alpha_e^2)}\,N^2 \tag{14}$$

times the impedance of a single hole. It is worth noting that the real impedance of N holes around the pipe at the same z is N^2 times the impedance of a single hole.

From Eq. (11) the loss factor for a Gaussian bunch of length σ_z is

$$k(\sigma_z)=\frac{Z_0 c\sqrt{\pi}}{128\pi^4 b^4\ \ln(d/b)\sigma_z^3}\Bigg[N^2(\alpha_m+\alpha_e)^2+N(\alpha_m-\alpha_e)^2 -2(\alpha_m-\alpha_e)^2\sum_{h=1}^{N-1}(N-h)e^{-(l^2/\sigma_z^2)h^2} \times\left(2\frac{l^2}{\sigma_z^2}h^2-1\right)\Bigg]. \tag{15}$$

The above expression is valid for bunch lengths $\sigma>(b+d)/2$. For shorter bunches, higher order modes have to be included in the theory.

A. Single hole

For a single hole, choosing the hole center as the origin of the longitudinal axis, system (8) becomes

$$\begin{pmatrix} 1+j\frac{\omega}{2}\alpha_m\mu h_{0\varphi}^2 & 0 \\ 0 & 1+j\frac{\omega}{2}\alpha_e\varepsilon e_{0r}^2 \end{pmatrix}\begin{pmatrix} M_\varphi(0) \\ P_r(0)\end{pmatrix} =\begin{pmatrix}\alpha_m H_{0\varphi}(0) \\ \varepsilon\alpha_e E_{0r}(0)\end{pmatrix}. \tag{16}$$

The real part of the longitudinal impedance is

$$Z_{\text{Re}}=\frac{Z_0 k_0^2}{16\pi^3 b^4\ \ln(d/b)}\,(\alpha_m^2+\alpha_e^2). \tag{17}$$

Replacing in Eq. (16) the polarizability for a round hole, one finds an impedance value five times larger than that previously presented in [3] which was affected by an oversight in the calculations. More recent results obtained by different methods [4] agree with Eq. (17).

From Eq. (15) the loss factor is

$$k(\sigma_z)=\frac{Z_0 c\sqrt{\pi}}{64\pi^4 b^4 \ln(d/b)\sigma_z^3}\,(\alpha_m^2+\alpha_e^2). \tag{18}$$

B. Two holes

Here we discuss the case of two holes, to better understand the interference and coupling effects. Choosing $z_1=0$ and $z_2=l$, the linear system for two holes becomes

$$\begin{pmatrix} 1+j\frac{\omega}{2}\alpha_m\mu h_{0\varphi}^2 & j\frac{\omega}{2}\alpha_m\mu h_{0\varphi}^2 e^{-jk_0 l} & 0 & -j\frac{\omega}{2}\alpha_m h_{0\varphi}e_{0r}e^{-jk_0 l} \\ j\frac{\omega}{2}\alpha_m\mu h_{0\varphi}^2 e^{-jk_0 l} & 1+j\frac{\omega}{2}\alpha_m\mu h_{0\varphi}^2 & j\frac{\omega}{2}\alpha_m h_{0\varphi}e_{0r}e^{-jk_0 l} & 0 \\ 0 & -j\frac{k_0\alpha_e}{2c}h_{0\varphi}e_{0r}e^{-jk_0 l} & 1+j\frac{\omega}{2}\alpha_e\varepsilon e_{0r}^2 & j\frac{\omega}{2}\alpha_e\varepsilon e_{0r}^2 e^{-jk_0 l} \\ j\frac{k_0\alpha_e}{2c}h_{0\varphi}e_{0r}e^{-jk_0 l} & 0 & j\frac{\omega}{2}\alpha_e\varepsilon e_{0r}^2 e^{-jk_0 l} & 1+j\frac{\omega}{2}\alpha_e\varepsilon e_{0r}^2 \end{pmatrix}\begin{pmatrix} M_\varphi(0) \\ M_\varphi(l) \\ P_r(0) \\ P_r(l)\end{pmatrix} =\begin{pmatrix}\alpha_m H_{0\varphi}(0) \\ \alpha_m H_{0\varphi}(l) \\ \varepsilon\alpha_e E_{0r}(0) \\ \varepsilon\alpha_e E_{0r}(l)\end{pmatrix}. \tag{19}$$

The real impedance, due to the interference between the propagating reaction fields, has the following approximate expression:

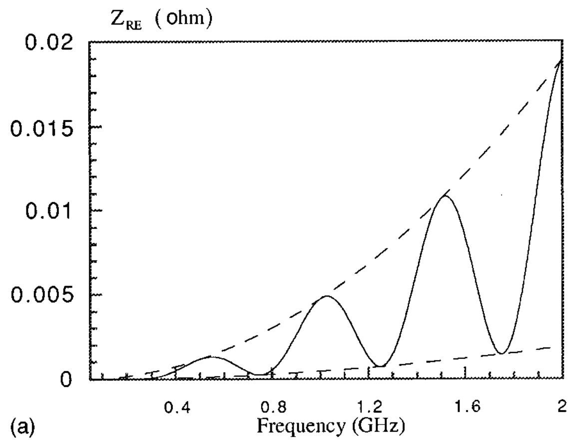
$$Z_{\text{Re}}=\frac{Z_0 k_0^2}{16\pi^3 b^4 \ln(d/b)}\{2(\alpha_m+\alpha_e)^2 +(\alpha_m-\alpha_e)^2[1+\cos(2k_0 l)]\}. \tag{20}$$

In Fig. 3(a) we show a typical plot of Z_{Re} for circular holes, as a function of the frequency. According to Eq. (14), the real part of the impedance oscillates between 4 and 0.4 times the single-hole value. Because of interference effects between the scattered fields in the coaxial pipe, maxima and minima occur at frequencies depending on the hole distance [Fig. 3(b)].

The loss factor, applying Eq. (15), is

$$k(\sigma_z)=\frac{Z_0 c\sqrt{\pi}}{64\pi^4 b^4 \ln(d/b)\sigma_z^3}\Bigg[2(\alpha_m+\alpha_e)^2+(\alpha_m-\alpha_e)^2 -(\alpha_m-\alpha_e)^2 e^{-(l^2/\sigma_z^2)}\left(2\frac{l^2}{\sigma_z^2}-1\right)\Bigg]. \tag{21}$$

In Fig. 4 (solid line) we show the loss factor for a $\sigma=5$ cm Gaussian bunch, for the same geometry of Fig. 3. The behavior of the loss factor is quite general, as we will see for



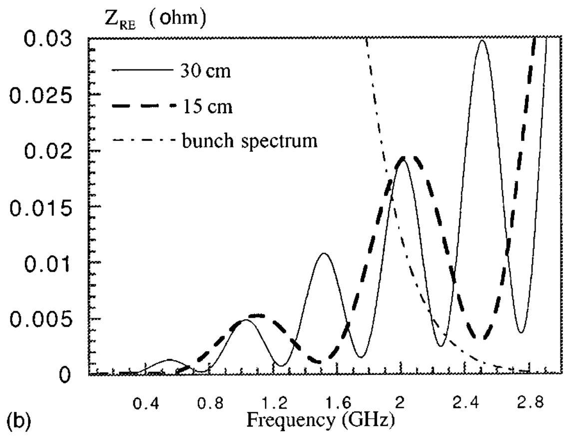


FIG. 3. (a) Z_{Re} for two round holes ($b=20$ mm, $d=24$ mm, $R=6$ mm, $l=300$ mm). (b) Z_{Re} for two round holes at different l, Gaussian bunch spectrum for $\sigma=50$ mm.

the case of N holes. It reaches a minimum value when $l\approx\sigma$, while it saturates for $l>3\sigma$. The minimum is caused by the destructive interference between fields, which surprisingly occurs only for one distance between the holes. For larger distances, the impedance has more maxima peaks under the bunch spectrum, however, since their amplitude decreases, the total area covered by the power spectrum remains almost constant.

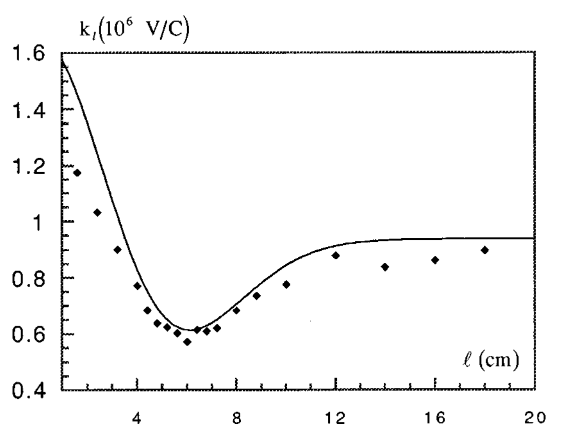


FIG. 4. Two-hole loss factor ($b=20$ mm, $d=24$ mm, $R=6$ mm, $\sigma=50$ mm).

C. Randomly spaced holes

It is interesting to compare the coupling impedance and the loss factor of N holes uniformly and randomly spaced. To calculate the impedance of N randomly spaced holes, we can assume in Eq. (11) $l_h=l+\delta_h$, where δ_h is a random variable. Again the imaginary part of the longitudinal impedance is N times the imaginary impedance of a single hole. The real part is

$$Z_{\mathrm{Re}}(\omega)=\frac{Z_0k_0^2}{16\pi^3b^4\ln(d/b)}\left\{\frac{N^2}{2}(\alpha_m+\alpha_e)^2+\frac{N}{2}(\alpha_m-\alpha_e)^2\right.$$
$$+(\alpha_m-\alpha_e)^2\sum_{h=1}^{N-1}\sum_{w=1}^{N-h}$$
$$\left.\times\cos\left[2k_0\left(wl+\sum_{t=1}^{w}\delta_{h+t}\right)\right]\right\}. \quad (22)$$

Consequently we can calculate the loss factor, which turns out to be

$$k(\sigma_z)=\frac{Z_0c\sqrt{\pi}}{128\pi^4b^4\ln(d/b)\sigma_z^3}\left\{N^2(\alpha_m+\alpha_e)^2+N(\alpha_m-\alpha_e)^2\right.$$
$$-2(\alpha_m-\alpha_e)^2\sum_{h=1}^{N-1}\sum_{w=1}^{h}e^{-(wl+\varepsilon_{N-h,w})^2/\sigma_z^2}$$
$$\left.\times\left[2\frac{(wl+\varepsilon_{N-h,w})^2}{\sigma_z^2}-1\right]\right\}, \quad (23)$$

where we have defined

$$\varepsilon_{N-h,w}=\sum_{t=N-h+1}^{N-h+w}\delta_t. \quad (24)$$

As an example, we compare the real part of the longitudinal impedance for 15 round holes with $l=30$ cm and δ_k uniformly distributed between $\pm0.2l$. We notice that the introduction of the positioning randomization clearly lowers the peak values (Fig. 5), while it does not affect the minima level. The loss factor, nevertheless, is almost unchanged (Fig. 6).

IV. COMPARISON OF ANALYTICAL AND NUMERICAL RESULTS

To check the validity of the expressions found, we performed simulations with the numerical code MAFIA [7] in the case of two holes [3]. To this end, it has been necessary to slightly modify the equations to account for the wall thickness which changes the problem geometry and introduces an attenuation for the fields in the holes.

Calling b_1 and b_2, respectively, the inner and the outer radius of the beam pipe, one can see that the factor b^4 in the denominator of Eqs. (12) and (15) has to be replaced by the product $b_1^2b_2^2$. Furthermore, the polarizabilities must be corrected; for a round hole of radius R we use the expressions in Ref. [8],

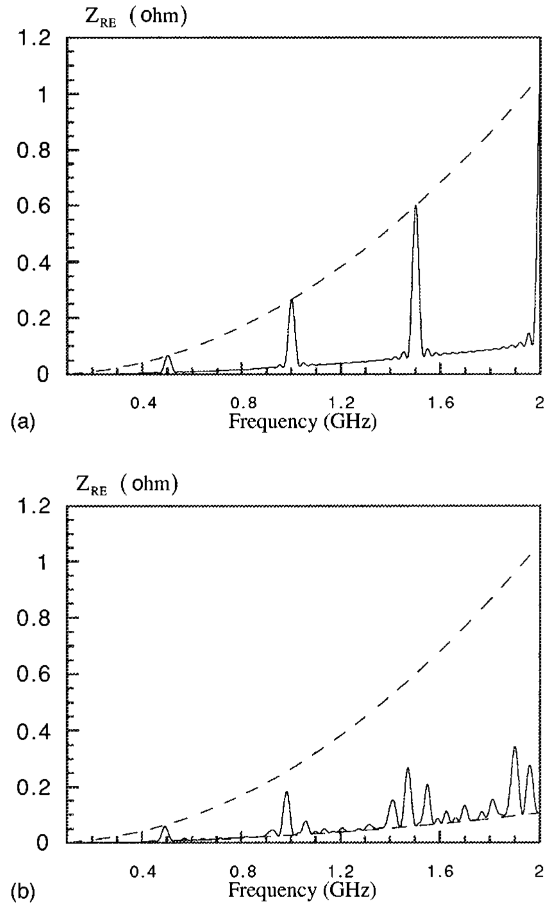


FIG. 5. (a) Z_{Re} for 15 round holes ($b=20$ mm, $d=24$ mm, $R=6$ mm, $l=300$ mm). (b) Z_{Re} for 15 round holes randomly spaced with uniform distribution $-0.2l \leqslant \delta_k \leqslant 0.2l$ ($b=20$ mm, $d=24$ mm, $R=6$ mm, $l=300$ mm).

$$\tilde{\alpha}_e = \alpha_e \frac{3.3}{4} e^{-\xi_{0,1} W/R},$$

$$\tilde{\alpha}_m = \alpha_m \frac{21}{25} e^{-\xi'_{1,1} W/R}, \tag{25}$$

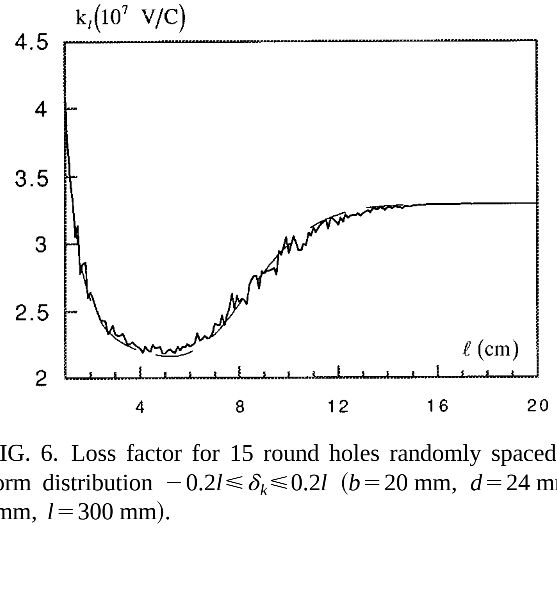


FIG. 6. Loss factor for 15 round holes randomly spaced with uniform distribution $-0.2l \leqslant \delta_k \leqslant 0.2l$ ($b=20$ mm, $d=24$ mm, $R=6$ mm, $l=300$ mm).

where W is the wall thickness (in our case $W=b_2-b_1$) and $\xi_{0,1}$ and $\xi'_{1,1}$ are the zeros of the Bessel function J_0 and J'_1, respectively.

We can thus rewrite Eq. (20) as

$$Z_{\mathrm{Re}} = \frac{Z_0 k_0^2}{16\pi^3 b_1^2 b_2^2 \ln(d/b_2)} \times \{2(\tilde{\alpha}_m+\tilde{\alpha}_e)^2 + (\tilde{\alpha}_m-\tilde{\alpha}_e)^2[1+\cos(2k_0 l)]\}. \tag{26}$$

As a result, the loss factor becomes

$$k(\sigma_z) = \frac{Z_0 c\sqrt{\pi}}{64\pi^4 b_1^2 b_2^2 \ln(d/b_2)\sigma_z^3}\left[2(\tilde{\alpha}_m+\tilde{\alpha}_e)^2 + (\tilde{\alpha}_m-\tilde{\alpha}_e)^2 - (\tilde{\alpha}_m-\tilde{\alpha}_e)^2 e^{-(l^2/\sigma_z^2)}\left(2\frac{l^2}{\sigma_z^2}-1\right)\right]. \tag{27}$$

In Fig. 4 the dependence of the loss factor on the hole distance l is presented for a $\sigma=5$ cm Gaussian bunch. The numerical results (black diamonds) are in good agreement with the analytical expression (solid line). The difference between theory and simulations tends to become larger for very short hole distances, when the coupling effect of the evanescent modes begins to be non-negligible.

V. CONCLUSIONS

The effect of the coupling between the equivalent dipoles seems to be important for a correct evaluation of the coupling impedance and the loss factor of N holes in a coaxial structure.

At low frequency, the real part of the longitudinal impedance grows as ω^2, as in the case of a single hole, being related to the TEM mode propagating in the coaxial region. Moreover, because of interference effects between the scattered fields, the real impedance and the loss factor are proportional to N^2.

A randomization in the hole position can lower significantly the peak value of the impedance while the minima and the loss factor are almost unchanged.

ACKNOWLEDGMENTS

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APPENDIX

A TEM field radiated by a hole centered in $z=z_i$ can be written as

$$E_r(z,z_i) = c_{0i} e_{0r} e^{-jk_0(z-z_i)} \theta(z-z_i) + d_{0i} e_{0r} e^{jk_0(z-z_i)} \theta(-z+z_i),$$

$$H_\varphi(z,z_i) = c_{0i} h_{0\varphi} e^{-jk_0(z-z_i)} \theta(z-z_i) - d_{0i} h_{0\varphi} e^{jk_0(z-z_i)} \theta(-z+z_i), \tag{A1}$$

where $k_0=\omega/c$, $\theta(z)$ is the Heaviside function, and

$$e_{0r}=\left(\frac{Z_0}{2\pi}\right)^{1/2}\frac{1}{\sqrt{\ln(d/b)}}\frac{1}{r},\quad h_{0\varphi}=\frac{1}{Z_0}e_{0r} \tag{A2}$$

are the normalized modal function for a TEM wave.

The coefficients c_{0i} and d_{0i} are given by

$$c_{0i}=\frac{j\omega}{2}[\mu h_{0\varphi}M_{\varphi}(z_i)+e_{0r}P_r(z_i)],$$

$$d_{0i}=-\frac{j\omega}{2}[\mu h_{0\varphi}M_{\varphi}(z_i)-e_{0r}P_r(z_i)]. \tag{A3}$$

When there are N holes radiating, the scattered fields on a generic hole center appearing in Eq. (1) are thus

$$E_{sr}(z_i)=e_{0r}\left[\sum_{k=1}^{i-1}c_{0k}e^{-jk_0(z_i-z_k)}+\frac{c_{0i}+d_{0i}}{2}+\sum_{k=i+1}^{N}d_{0k}e^{jk_0(z_i-z_k)}\right],$$

$$H_{s\varphi}(z_i)=h_{0\varphi}\left[\sum_{k=1}^{i-1}c_{0k}e^{-jk_0(z_i-z_k)}+\frac{c_{0i}-d_{0i}}{2}-\sum_{k=i+1}^{N}d_{0k}e^{jk_0(z_i-z_k)}\right]. \tag{A4}$$

Replacing Eq. (A3) in Eq. (A4) one obtains Eqs. (6) and (7).

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